# Sample Question Paper - 3
## CLASS: XII
## Session: 2021-22
## Mathematics (Code-041)
## Term - 1

**Time Allowed: 1 hour and 30 minutes** **Maximum Marks: 40**

**General Instructions:**

1. This question paper contains three sections – A, B and C. Each part is compulsory.
2. Section - A has 20 MCQs, attempt any 16 out of 20. 3
3. . Section - B has 20 MCQs, attempt any 16 out of 20
4. Section - C has 10 MCQs, attempt any 8 out of 10.
5. There is no negative marking.
6. All questions carry equal marks.

**SECTION – A**

**Attempt any 16 questions**

1. Let A = { 2 , 3 , 6 }. Which of the following relations on A are reflexive? **[1]**

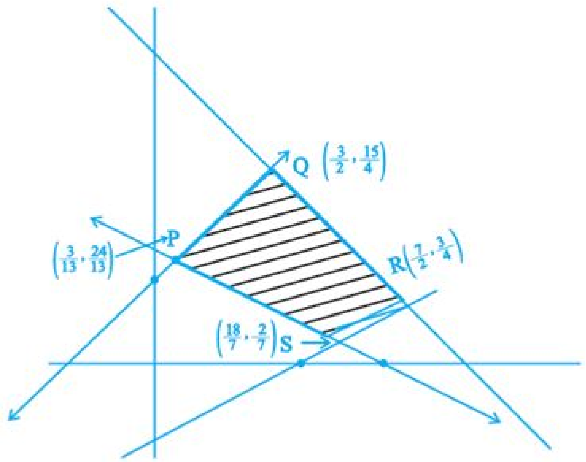
a) None of these

b) $R_1$ = {( 2,2 ) , ( 3,3 ) , ( 6,6 )}

c) $R_2$ = {( 2,2 ) , ( 3,3 ) , ( 3,6 ) , ( 6,3 )}

d) $R_3$ = {( 2,2 ) , ( 3,6 ) , ( 2,6 )}

2. In Figure, the feasible region (shaded) for a LPP is shown. Determine the maximum and minimum value of Z = x + 2y **[1]**




a) Maximum = 10, minimum = $3\frac{1}{4}$

b) Maximum = 8, minimum = $3\frac{1}{6}$

c) Maximum = 7, minimum = $3\frac{1}{9}$

d) Maximum = 9, minimum = $3\frac{1}{7}$

3. If x = $a\cos^3 t$, y = $a\sin^3 t$, then $\frac{dy}{dx}$ is equal to **[1]**

a) – tan t

b) cosec t

c) cos t

d) cot t

**[1]**

4. The function f(x) = cot $^{-1}$ x +x increases in the interval

a) 0 , ∞

b) -∞ ,∞

c) (1, ∞)

d) -1, ∞

5. The point at which the maximum value of x + y, subject to the constraints x + 2y $\leq$ 70, 2x + y $\leq$ 95, x, y $\geq$ 0 is obtained, is **[1]**

a) (20, 35)

b) (30, 25)

c) (35, 20)

d) (40,15)

6. The system of equations, x + y = 2 and 2x + 2y = 3 has **[1]**

a) a unique solution

b) finitely many solutions

c) no solution

d) infinitely many solutions

7. If y = $x^{x\sin x}$ then $\frac{dy}{dx} = ?$ **[1]**

a) $x^{\sin x}\left\{\frac{\sin x+x\log x\cdot\sin x}{x}\right\}$

b) $(\sin x\cos x)\cdot x^{(\sin x-1)}$

c) None of these

d) $(\sin x)\cdot x^{(\sin x-1)}$

8. If A and B are matrices of same order, then (AB' – BA') is a **[1]**

a) null matrix

b) unit matrix

c) symmetric matrix

d) skew-symmetric matrix

9. Maximize Z = 3x + 4y, subject to the constraints : x + y ≤ 1, x ≥ 0, y ≥ 0. **[1]**

a) 4

b) 5

c) 6

d) 3

10. If $y=\sqrt{\frac{1+\tan x}{1-\tan x}}$ then $\frac{dy}{dx}=?$ **[1]**

a) $\frac{\sec^2\left(\frac{x}{4}\right)}{\sqrt{\tan\left(x+\frac{\pi}{4}\right)}}$

b) $\frac{1}{2}\sec^2 x\cdot\tan\left(x+\frac{\pi}{4}\right)$

c) $\frac{\sec^2\left(x+\frac{\pi}{4}\right)}{2\sqrt{\tan\left(x+\frac{\pi}{4}\right)}}$

d) none of these

11. Let $f(x)=\begin{cases}\frac{1}{|x|} & \text{for } |x|\geq 1\\ ax^2+b & \text{for } |x|<1\end{cases}$ If f(x) is continuous and differentiable at any point, then **[1]**

a) a = 1, b = –1

b) $a=\frac{1}{2}, b=-\frac{3}{2}$

c) a $=\frac{1}{2}$, b $=\frac{3}{2}$

d) none of these

12. Minimize Z = 5x + 10 y subject to x + 2y ≤ 120, x + y ≥ 60, x – 2y ≥ 0, x, y ≥ 0 **[1]**

a) Minimum Z = 310 at (60, 0)

b) Minimum Z = 320 at (60, 0)

c) Minimum Z = 330 at (60, 0)

d) Minimum Z = 300 at (60, 0)

13. The normal to the curve x = a $(\cos\theta+\theta\sin\theta)$ ,y = a $(\sin\theta-\theta\cos\theta)$ at any point $\theta$ is such that **[1]**

a) it is at a constant distance from the origin

b) it passes through the origin

c) it makes a constant angle with X – axis

d) none of these

14. The function f(x) = $\sin^{-1}(\cos x)$ is **[1]**

a) None of these

b) differentiable at x = 0

c) discontinuous at x = 0

d) continuous at x = 0

15. The equation of the tangent to the curve y = x log x is parallel to the chord joining the points (1, 0) and (e, e), the value of x is: **[1]**

a) $e^{1/1-e}$

b) $e^{(e-1)(2e-1)}$

c) $e^{\frac{2e-1}{e-1}}$

d) $\frac{e-1}{e}$

16. Assume X, Y, Z, W, and P are matrices of order 2 $\times$ n, 3 $\times$ k, 2 $\times$ p, n $\times$ 3 and p $\times$ k, respectively. **[1]**

The restriction on n, k and p so that PY + WY will be defined are

a) p is arbitrary, k = 3

b) k is arbitrary, p = 2

c) k = 2, p = 3

d) k = 3, p = n

17. At what points the slope of the tangent to the curve $x^2 + y^2 - 2x - 3 = 0$ is zero **[1]**

a) (3, 0), (1, 2)

b) (-1, 0), (1, 2)

c) (3, 0), (-1, 0)

d) (1, 2), (1, -2)

18. If $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$, then x = **[1]**

a) $\frac{1}{2}$

b) None of these

c) $\frac{\sqrt{3}}{2}$

d) $-\frac{1}{2}$

19. If $f(x) = \sqrt{x^2 + 6x + 9}$, then f'(x) is equal to **[1]**

a) 1 for all x $\in$ R

b) none of these

c) 1 for x < -3

d) -1 for x < -3

20. If A is a square matrix, then AA is a **[1]**

a) none of these

b) skew-symmetric matrix

c) symmetric matrix

d) diagonal matrix

**SECTION – B**

**Attempt any 16 questions**

21. Let $f : R \rightarrow R$ be defined by f (x) = $\frac{1}{x}$ , $\forall x \in R$ . Then f is **[1]**

a) one – one

b) Bijective

c) f is not defined

d) Onto

22. The minimum value of f(x) = $3x^4 - 8x^3 - 48x + 25$ on [0, 3] is **[1]**

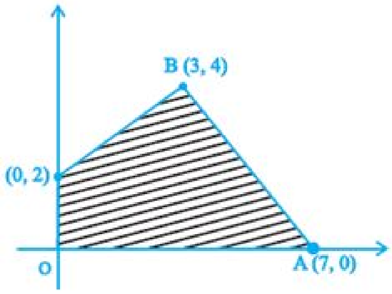
a) 25

b) 16

c) -39

d) None of these

23. Feasible region (shaded) for a LPP is shown in Figure. Maximize Z = 5x + 7y. **[1]**




a) 45

b) 49

c) 47

d) 43

24. If y = $\cos^2 x^3$ then $\frac{dy}{dx} = ?$ **[1]**

a) $-3x^2 \sin^2 x^3$

b) none of these

c) $-3x^2 \cos^2 (2x^3)$

d) $-3x^2 \sin (2x^3)$

25. If y = $ax^2 + bx + c$, then $y^3 \frac{d^2y}{dx^2}$ is **[1]**

a) a constant

b) a function of x only

c) a function of y only

d) a function of x and y

26. $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$ is equal to **[1]**

a) $\frac{1}{4}$

b) $\frac{1}{3}$

c) 1

d) $\frac{1}{2}$

27. R is a relation on the set Z of integers and it is given by (x, y) $\in$ R $\Leftrightarrow$ $|x - y| \leq 1$. Then, R is **[1]**

a) an equivalence relation

b) symmetric and transitive

c) reflexive and symmetric

d) reflective and transitive

28. $\sin^{-1}(1 - x) - 2\sin^{-1} x = \frac{\pi}{2}$ then x is equal to **[1]**

a) $\frac{1}{2}$

b) $(0, \frac{1}{2})$

c) $(1, \frac{1}{2})$

d) 0

29. If A + B = $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and A – 2 B = $\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$, then A = ? **[1]**

a) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$

b) none of these

c) $\frac{1}{3}\begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}$

d) $\frac{1}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

30. If y = $\tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$ then $\frac{dy}{dx} = ?$ **[1]**

a) 1

b) $\frac{1}{2}$

c) -1

d) $\frac{-1}{2}$

31. If f(x) = $|\log_e x|$, then **[1]**

a) f’(1) = –1

b) f’(1) = 1

c) $f'(1^-) = -1$

d) $f'(1^+) = 1$

32. If $y = \log\sqrt{\tan x}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ is given by **[1]**

a) 0

b) $\infty$

c) $\frac{1}{2}$

d) 1

33. The function f(x) = $x^9 + 3x^7 + 64$ is increasing on **[1]**

a) $(-\infty, 0)$

b) $R_0$

c) $(0, \infty)$

d) R

34. If $\cos^{-1}x > \sin^{-1}x$, then **[1]**

a) $0 \le x < \frac{1}{\sqrt{2}}$

b) $\frac{1}{\sqrt{2}} < x \le 1$

c) $-1 \le x < \frac{1}{\sqrt{2}}$

d) x > 0

35. Any tangent to the curve y = $2x^7 + 3x + 5$ **[1]**

a) is parallel to x-axis

b) is parallel to y-axis

c) makes an acute angle with x-axis

d) makes an obtuse angle with x-axis

36. In a LPP, the linear inequalities or restrictions on the variables are called **[1]**

a) Limits

b) Inequalities

c) Linear constraints

d) Constraints

37. If A is an invertible square matrix and k is a non-negative real number then$(kA)^{-1}$ = ? **[1]**

a) $\frac{1}{k} \cdot A^{-1}$

b) $-k.A^{-1}$

c) $k.A^{-1}$

d) None of these

38. If a matrix A is symmetric as well as skew symmetric, then A is a **[1]**

a) none of these

b) null matrix

c) unit matrix

d) diagonal matrix

39. If $\sqrt{x} + \sqrt{y} = \sqrt{a}$, then $\left(\frac{d^2y}{dx^2}\right)_{x=a}$ is equal to **[1]**

a) $\frac{1}{2a}$

b) a

c) None of these

d) $\frac{1}{a}$

40. Let f : R $\rightarrow$ R be a function defined by $f(x) = \frac{x^2-8}{x^2+2}$. Then, f is **[1]**

a) one-one and onto

b) one-one but not onto

c) onto but not one-one

d) neither one-one nor onto

## SECTION – C

**Attempt any 8 questions**

41. If $\alpha = \tan^{-1}\left(\tan\frac{5\pi}{4}\right)$ and $\beta = \tan^{-1}\left(-\tan\frac{2\pi}{3}\right)$, then **[1]**

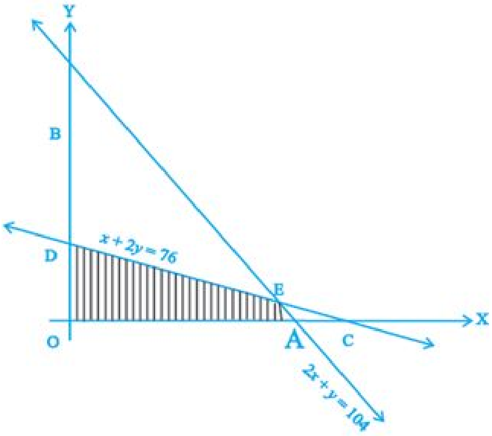
a) none of these

b) $\alpha - \beta = \frac{7\pi}{12}$

c) $3a = 4\beta$

d) $4a = 3\beta$

42. Determine the minimum value of Z = 3x + 4y if the feasible region (shaded) for a LPP is shown in Figure above. **[1]**




a) 154

b) 196

c) 112

d) 132

43. If $y = ae^{mx} + be^{-mx}$, then $y_2$ is equal to **[1]**

a) $my_1$

b) $-m^2y$

c) $m^2y$

d) None of these

44. Let $f(x) = 2x^3 - 3x^2 - 12x + 5$ on [-2, 4]. The relative maximum occurs at x = **[1]**

a) 2

b) -1

c) 4

d) -2

45. S is a relation over the set R of all real numbers and its is given by $(a, b) \in S \Leftrightarrow ab \geq 0$. Then, S is **[1]**

a) an equivalence relation

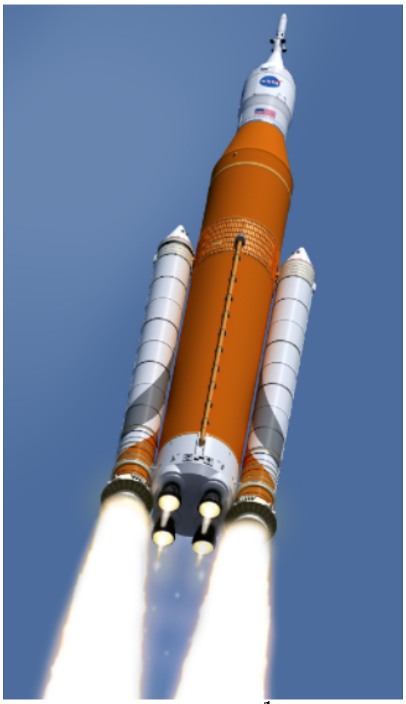
b) reflexive and symmetric only

c) symmetric and transitive only

d) antisymmetric relation

**Question No. 46 to 50 are based on the given text. Read the text carefully and answer the questions:**

The upward speed v(t) of a rocket at time t is approximated by $v(t) = at^2 + bt + c$, $0 \leq t \leq 100$, where a, b and c are constants. It has been found that the speed at times t = 3, t = 6 and t = 9 seconds are respectively 64, 133 and 208 miles per second.



If $\begin{pmatrix} 9 & 3 & 1 \\ 36 & 6 & 1 \\ 81 & 9 & 1 \end{pmatrix}^{-1} = \frac{1}{18}\begin{pmatrix} 1 & -2 & 1 \\ -15 & 24 & -9 \\ 54 & -54 & 18 \end{pmatrix}$,

46. v(t) is given by **[1]**

a) $t^2 + \frac{1}{3}t + 20$
b) $t^2 + 20t + 1$
c) $t^2 + t + 1$
d) $\frac{1}{3}t^2 + 20t + 1$

47. The speed at time t = 15 seconds is **[1]**

a) 366 miles/sec
b) 346 miles/sec
c) 376 miles/sec
d) 356 miles/sec

48. The time at which the speed of rocket is 784 miles/sec is **[1]**

a) 20 seconds
b) 25 seconds
c) 30 seconds
d) 27 seconds

49. The value of b + c is **[1]**

a) 21
b) $\frac{3}{4}$
c) $\frac{4}{3}$
d) 20

50. The value of a + c is **[1]**

a) 1
b) none of these
c) $\frac{4}{3}$
d) 20

# Solution

## SECTION – A

1. **(b)** $R_1$ = {( 2,2 ) , ( 3,3 ) , ( 6,6 )}

   **Explanation:** $R_1$ is a reflexive on A, because ( a,a ) $\in R_1$ for each $a \in A$

2. **(d)** Maximum = 9, minimum = $3\frac{1}{7}$

   **Explanation:**

| Corner points | Z = x +2 y |
|---|---|
| P(3/13,24/13 ) | 51/13 |
| Q(3/2,15/4) | 9.......................(Max.) |
| R(7/2,3/4 ) | 5 |
| S(18/7,2/7) | 22/7.......................(Min.) |

   Hence the maximum value is 9 and the minimum value is $3\frac{1}{7}$

3. **(a)** – tan t

   **Explanation:** We have to find: $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3a\sin^2 t\cos t}{3a\cos^2 t(-\sin t)} = -\tan t$

4. **(b)** -∞ ,∞

   **Explanation:** $(-\infty, \infty)$

   $f(x) = \cot^{-1} x + x$

   $f'(x) = \frac{-1}{1+x^2} + 1$

   $= \frac{-1+1+x^2}{1+x^2}$

   $= \frac{x^2}{1+x^2} \geq 0, \forall x \in R$

   So, f (x) is increasing on $(-\infty, \infty)$

5. **(d)** (40,15)

   **Explanation:** We need to maximize the function z = x + y Converting the given inequations into equations, we obtain

   x + 2y = 70, 2x + y = 95, x = 0 and y = 0

   Region represented by x + 2y $\leq$ 70 :

   The line x + 2y = 70 meets the coordinate axes at A(70, 0) and B(0, 35) respectively. By joining these points we obtain the line x + 2y = 70. Clearly (0, 0) satisfies the inequation x + 2y $\leq$ 70 . So, the region containing the origin represents the solution set of the inequation x + 2y $\leq$ 70.

   Region represented by 2x + y $\leq$ 95 :

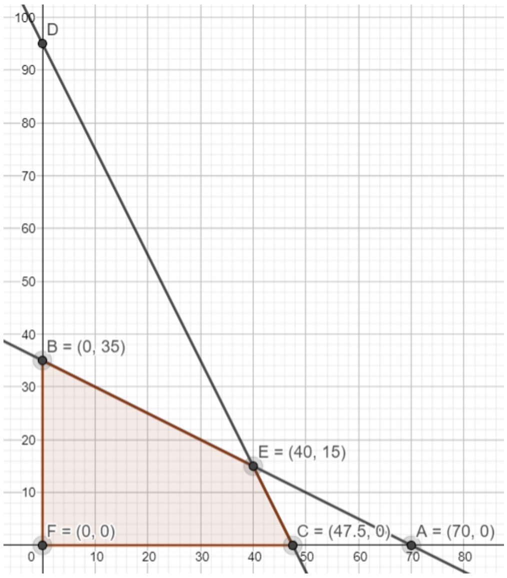
   The line 2x + y = 95 meets the coordinate axes at $C\left(\frac{95}{2}, 0\right)$ respectively. By joining these points we obtain the line 2x + y = 95

   Clearly (0, 0) satisfies the inequation 2x + y $\leq$ 95 . So, the region containing the origin represents the solution set of the inequation 2x + y $\leq$ 95

   Region represented by x $\geq$ 0 and y $\geq$ 0 :

   since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations x $\geq$ 0, and y $\geq$ 0

   The feasible region determined by the system of constraints x + 2y $\leq$ 70, 2x + y $\leq$ 95, x $\geq$ 0, and y $\geq$ 0 are as follows.




The corner points of the feasible region are O(0, 0), C($\frac{95}{2}$, 0) E(40, 15) and B(0, 35).

The value fo Z at these corner points are as follows.

Corner point : z = x + y

O(0, 0) : 0 + 0 = 0

$C\left(\frac{95}{2},0\right):\frac{95}{2}+0=\frac{95}{2}$

E(40, 15) : 40 + 15 = 55

B(0, 35) : 0 + 35 = 35

We see that maximum value of the objective function Z is 55 which is at (40, 15).

6. **(c)** no solution

**Explanation:** For No solution, $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$, for given system of equations we have:$\frac{1}{2}=\frac{1}{2}\neq\frac{2}{3}$.

7. **(a)** $x^{\sin x}\left\{\frac{\sin x+x\log x\cdot\sin x}{x}\right\}$

**Explanation:** Let y = f(x) = $x^{\sin x}$

Taking log both sides, we obtain

$\log_e y = \sin x \log_e x$ -(1) (Since $\log_a b^c = c \log_a b$)

Differentiating (i) with respect to x, we obtain

$\frac{1}{y}\frac{dy}{dx}=\sin x\times\frac{1}{x}+\log_e x\times\cos x$

$\Rightarrow\frac{dy}{dx}=y\times\left(\frac{\sin x}{x}+\log_e x\cos x\right)$

$\Rightarrow\frac{dy}{dx}=f'(x)=x^{sinx}\left(\frac{\sin x+x\log x\sin x}{x}\right)$.

Which is the required solution.

8. **(d)** skew-symmetric matrix

**Explanation:** We have matrices A and B of same order.

Let P = (AB' − BA')

Then, P' = (AB' − BA')'

= (AB')' − (BA')'

= (B')' (A)' − (A')'B' = BA' − AB' = −(AB' − BA') = −P

Therefore, the given matrix (AB - BA') is a skew-symmetric matrix.

9. **(a)** 4

**Explanation:** According to the question, maximize , Z = 3x + 4y, subject to the constraints: x + y ≤ 1, x ≥ 0, y ≥ 0.

| Corner points | Z = 3x +4 y |
|---|---|
| C(0, 0 ) | 0 |
| B (1,0) | 3 |
| D(0,1 ) | 4 |

Hence the maximum value is 4

10. **(c)** $\frac{\sec^2\left(x+\frac{\pi}{4}\right)}{2\sqrt{\tan\left(x+\frac{\pi}{4}\right)}}$

**Explanation:** Given that $y = \sqrt{\frac{1+\tan x}{1-\tan x}}$

Using $\tan\left(\frac{\pi}{4} + x\right) = \frac{1+\tan x}{1-\tan x}$, we obtain

$y = \sqrt{\tan\left(\frac{\pi}{4} + x\right)}$

Differentiating with respect to x, we obtain

$\frac{dy}{dx} = \frac{1}{2\sqrt{\tan\left(\frac{\pi}{4}+x\right)}} \times \sec^2\left(\frac{\pi}{4} + x\right) \times 1$

Hence, $\frac{dy}{dx} = \frac{\sec^2\left(\frac{\pi}{4}+x\right)}{2\sqrt{\tan\left(\frac{\pi}{4}+x\right)}}$

11. **(d)** none of these

**Explanation:** Given that $f(x) = \left\{\begin{array}{c} \frac{-1}{x}, x \leq -1 \\ ax^2 + b, -1 < x < 1 \\ \frac{1}{x}, x \geq 1 \end{array}\right\}$

∵ f(x) is continuous and differentiable at any point, consider x =1.

$\lim\limits_{x\to1} \frac{1}{x} = \lim_{x\to1} ax^2 + b$

⇒a + b = 1

Also,

$\Rightarrow \lim\limits_{x\to1^-} \frac{f(x)-f(1)}{x-1} = \lim\limits_{x\to1^+} \frac{f(x)-f(1)}{x-1}$

$\Rightarrow \lim\limits_{x\to1} \frac{ax^2-a}{x-1} = \lim \frac{1-x}{x(x-1)}$

$\Rightarrow \lim\limits_{x\to1} a(x+1) = \lim\limits_{x\to1}(-x)$

$\Rightarrow a = \frac{-1}{2}$

Putting above value in a + b = 1,we get

$\mathrm{b} = \frac{3}{2}$.

Which is the required value of a and b.

12. **(d)** Minimum Z = 300 at (60, 0)

**Explanation:** Objective function is Z = 5x + 10 y .......................(1).

The given constraints are : x + 2y ≤ 120, x + y ≥ 60, x – 2y ≥ 0, x, y ≥ 0 .

The corner points are obtained by drawing the lines x+2y =120, x+y = 60 and x-2y = 0. The points so obtained are (60,30),(120,0), (60,0) and (40,20)

| Corner points | Z = 5x + 10y |
|---|---|
| D(60 ,30 ) | 600 |
| A(120,0) | 600 |
| B(60,0) | 300.........................(Min.) |
| C(40,20) | 400 |

Here , Z = 300 is minimum at ( 60, 0 ).

13. **(a)** it is at a constant distance from the origin

**Explanation:** Equation of normal at $\theta \quad is\ x cos\theta + y sin\theta - a = 0$. So,normal is at a fixed distance a from the origin.

14. **(d)** continuous at x = 0

**Explanation:** Given f(x) = $\sin^{-1}(\cos x)$,

Checking differentiability and continuity,

LHL at x = 0,

$$\lim_{\mathbf{x}\to 0^-} \mathbf{f}(\mathbf{x}) = \lim_{\mathbf{h}\to 0} \mathbf{f}(0-\mathbf{h}) = \lim_{\mathbf{h}\to 0} \sin^{-1}(\cos(0-\mathbf{h})) = \lim_{\mathbf{h}\to 0} \sin^{-1}(\cos(-\mathbf{h})) = \sin^{-1} 1 = \frac{\pi}{2}$$

RHL at x =0,

$$\lim_{\mathbf{x}\to 0^+} \mathbf{f}(\mathbf{x}) = \lim_{\mathbf{h}\to 0} \mathbf{f}(0+\mathbf{h}) = \lim_{\mathbf{h}\to 0} \sin^{-1}(\cos(0+\mathbf{h})) = \lim_{\mathbf{h}\to 0} \sin^{-1}(\cos(\mathbf{h})) = \sin^{-1} 1 = \frac{\pi}{2}$$

And $f(0) = \frac{\pi}{2}$

Hence, f(x) is continuous at x =0.

LHD at x = 0,

$$\lim_{x\to 0^-} \frac{f(x)-f(0)}{x-0} = \lim_{h\to 0} \frac{f(0-h)-f(0)}{0-h-0}$$

$$= \lim_{h\to 0} \frac{\sin^{-1}(\cos(0-h))-\left(\frac{\pi}{2}\right)}{-h} = 1$$

RHD at x =0,

$$\lim_{x\to 0^+} \frac{f(x)-f(0)}{x-0} = \lim_{h\to 0} \frac{f(0+h)-f(0)}{0+h-0}$$

$$= \lim_{h\to 0} \frac{\sin^{-1}(\cos(0+h))-\left(\frac{\pi}{2}\right)}{h} = -1$$

∵ LHD $\neq$ RHD

∴ f(x) is not differentiable at x =0.

15. **(a)** $e^{1/1-e}$

**Explanation:** y = x log x

Differentiating the function with respect to ‘x’,

$$\frac{dy}{dx} = 1 + \log x$$

Slope of tangent to the curve = 1 + log x

And, slope of the chord joining the points, (1, 0) & (e, e)

$$\mathrm{m} = \frac{\mathrm{e}}{\mathrm{e}-1}$$

The tangent to the curve is parallel to the chord joining the points, (1, 0) & (e, e)

∴ m = 1 + log x

$$\frac{e}{e-1} = 1 + \log x$$

$$\log x = \frac{e}{e-1} - 1$$

$$\log x = \frac{e-e+1}{e-1}$$

$$\log x = \frac{1}{e-1}$$

$$x = e^{\frac{1}{1-e}}$$

16. **(d)** k = 3, p = n

**Explanation:** Matrices P and Y are of the orders p $\times$ k and 3 $\times$ k respectively.

Therefore, matrix PY will be defined if k = 3.

Then, PY will be of the order p $\times$ k = p$\times$ 3.

Matrices W and Y are of the orders n $\times$ 3 and 3 $\times$ k = 3 $\times$ 3 respectively.

As, the number of columns in W is equal to the number of rows in Y, Matrix WY is well defined and is of the order n $\times$ 3.

Matrices PY and WY can be added only when their orders are the same.

Therefore, PY is of the order p $\times$ 3 and WY is of the order n $\times$ 3.

Thus, we must have p = n.

Therefore, k = 3 and p = n are the restrictions on n, k and p so that PY + WY will be defined.

17. **(d)** (1, 2), (1, -2)

**Explanation:** $x^2 + y^2 - 2x - 3 = 0$

Differentiating with respect to x,

$2x + 2y\frac{dy}{dx} - 2 = 0$

$\Rightarrow \frac{dy}{dx} = \frac{2-2x}{2y}$

Given that slope of tangent $= \frac{dy}{dx} = 0$

$\Rightarrow \frac{2-2x}{2y} = 0$

x = 1

$x^2 + y^2 - 2x - 3 = 0$

$\Rightarrow y^2 = 2x + 3 - x^2$

x = 1

$\Rightarrow y = \pm 2$

Point are (1, 2) and (1, -2)

18. **(c)** $\frac{\sqrt{3}}{2}$

**Explanation:** $\sin^{-1} - \cos^{-1} x = \frac{\pi}{6}$

$\frac{\pi}{2} - \cos^{-1} x - \cos^{-1} x = \frac{\pi}{6}$

$\frac{\pi}{2} - 2\cos^{-1} x = \frac{\pi}{6}$

$\frac{\pi}{2} - \frac{\pi}{6} = 2\cos^{-1} x$

$\frac{2\pi}{6} = 2\cos^{-1} x$

$\frac{\pi}{3} \times \frac{1}{2} = \cos^{-1} x$

$\frac{\pi}{6} = \cos^{-1} x$

$x = \cos\frac{\pi}{6}$

$x = \frac{\sqrt{3}}{2}$

19. **(d)** -1 for x < -3

**Explanation:** We have, $f(x) = \sqrt{x^2 + 6x + 9}$

$= \sqrt{(x+3)^2}$

$= |x + 3|$

$f(x) = \begin{cases} x+3 & x \geq -3 \\ -x-3 & x < -3 \end{cases}$

$\Rightarrow f'(x) = \begin{cases} 1 & x \geq -3 \\ -1 & x < -3 \end{cases}$

$\therefore f'(x) = -1$ for $x < -3$.

Which is the required solution.

20. **(a)** none of these

**Explanation:** If A is a square matrix,

Let A = $\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$

AA = $\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$

then AA is neither of the matrices given in the options of the question.

## SECTION – B

21. **(c)** f is not defined

**Explanation:** Because , $\frac{1}{x}$ $is\ not\ defined\ for\ x = 0, as\ 0 \in R, \therefore f\ is\ not\ defined.$

22. **(c)** -39

**Explanation:** Given function,

$f(x) = 3x^4 - 8x^3 - 48x + 25$

$F'(x) = 12x^3 - 24x^2 - 48 = 0$

$F'(x) = 12(x^3 - 2x^2 - 4) = 0$

Differentiating again, we obtain

F''(x) = $3x^2 - 4x = 0$
$x(3x - 4) = 0$
x = 0 or x = $\frac{4}{3}$
Putting the value in equation, we obtain
f(x) = -39

23. **(d)** 43
**Explanation:**

| Corner points | Z = 5x + 7y |
|---|---|
| O(0,0) | 0 |
| B (3,4) | 43 |
| A(7,0) | 35 |
| C(0,2) | 14 |

Hence the maximum value is 43

24. **(d)** $-3x^2 \sin(2x^3)$
**Explanation:** Given, $y = \cos^2 x^3 = (\cos(x^3))^2$
$\frac{dy}{dx} = (2\cos x^3)(-\sin(x^3)) \times 3x^2$
Using 2 sin A cos A = sin 2A
$\frac{dy}{dx} = -3x^2 \sin(2x^3)$

25. **(c)** a function of y only
**Explanation:** $y = ax^2 + bx + c$
$\frac{dy}{dx} = 2ax + b$
$\frac{d^2y}{dx^2} = 2a$
$y^3\frac{d^2y}{dx^2} = 2ay^3$ = A function of y only

26. **(c)** 1
**Explanation:** $\sin^{-1}\left(-\frac{1}{2}\right) = -\sin^{-1}\left(\frac{1}{2}\right)$, as $sin^{-1}(-x) = -sin^{-1}x$
We all know that the principle branch of $sin^{-1}x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $-\frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\therefore \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$
Now, $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right) = \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right) = \sin\left(\frac{3\pi}{6}\right) = \sin\left(\frac{\pi}{2}\right) = 1$
Therefore, the required value of $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right) = 1$

27. **(c)** reflexive and symmetric
**Explanation:** According to the condition,
$(x,y) \in R \implies |x - y| \leq 1$
Reflexive: let $(x,x) \in R \implies |x-x| = 0 < 1$
$\Rightarrow$R is Reflexive
Symmetric:
If $(x,y) \in R \implies |x-y| \leq 1$
and $(y,x) \in R \implies |y-x| \leq 1$ [Since $|x-y| = |y-x|$]
$\Rightarrow$R is Symmetric
Transitive:
If $(x,y) \in R \Rightarrow |x-y| \leq 1$
and $(y,z) \in R \Rightarrow |y-z| \leq 1$
$\Rightarrow |x-y| = |x-y+y-z|$
$\leq |x-y| + |y-z| \leq 1+1 = 2$
$\Rightarrow |x-z| \leq 2$
$\therefore$R is not transitive

28. **(d)** 0

**Explanation:** $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$

Now, we will put x = sin y in the given equation, and we get

$\sin^{-1}(1-\sin y)-2\sin^{-1}\sin y=\frac{\pi}{2}$

$\Rightarrow \sin^{-1}(1-\sin y)-2y=\frac{\pi}{2}$

$\Rightarrow \sin^{-1}(1-\sin y)=\frac{\pi}{2}+2y$

$\Rightarrow 1-\sin y=\sin\left(\frac{\pi}{2}+2y\right)$

$\Rightarrow 1-\sin y=\cos 2y(as \sin\left(\frac{\pi}{2}+x\right)=\cos x)$

$\Rightarrow 1-cos2y=siny$

$\Rightarrow 2\ sin2y=siny$

$\Rightarrow siny.(2siny-1)=0$

$\Rightarrow siny=0\ \ or\ \ siny=\frac{1}{2}$

$\therefore\ \ x=0\ \ or\ \ x=\frac{1}{2}$

Now, if we put x = $\frac{1}{2}$, then we will see that,

L.H.S. = $\sin^{-1}\left(1-\frac{1}{2}\right)-2\sin^{-1}\frac{1}{2}$

= $\sin^{-1}\left(\frac{1}{2}\right)-2\sin^{-1}\frac{1}{2}$

= $-\sin^{-1}\frac{1}{2}$

= $-\frac{\pi}{6}\neq\frac{\pi}{2}\neq$ R.H.S

Hence, x = $\frac{1}{2}$ is not the solution of the given equation.

Thus, x = 0

29. **(c)** $\frac{1}{3}\begin{bmatrix}1 & 1\\ 3 & 1\end{bmatrix}$

**Explanation:** A + B = $\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$ ...(i)

A - 2B = $\begin{bmatrix}-1 & 1\\ 1 & -1\end{bmatrix}$ ...(ii)

adding 2 $\times$ (i) and (ii), we get

2A + 2B = $\begin{bmatrix}2 & 0\\ 2 & 2\end{bmatrix}$ ...(iii)

A - 2B = $\begin{bmatrix}-1 & 1\\ 1 & -1\end{bmatrix}$ ...(iv)

adding (iii) and (iv), we get

$\Rightarrow$ 3A = $\begin{bmatrix}2 & 0\\ 2 & 2\end{bmatrix}+\begin{bmatrix}-1 & 1\\ 1 & -1\end{bmatrix}$

$\Rightarrow$ A = $\frac{1}{3}\begin{bmatrix}1 & 1\\ 3 & 1\end{bmatrix}$

30. **(b)** $\frac{1}{2}$

**Explanation:** Given that y = $\tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$

Using 1 - cos x = 2sin$^2\frac{x}{2}$ and Using sin x = 2 sin x $\frac{x}{2}$cos $\frac{x}{2}$, we obtain

y = $\tan^{-1}\left(\frac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right)$ or y = $\tan^{-1}\tan\frac{x}{2}$

$y=\frac{x}{2}$

Differentiating with respect to x, we obtain

$\frac{dy}{dx}=\frac{1}{2}$

31. **(c)** f'(1$^-$) = –1

**Explanation:** Given that $f(x)=\begin{Bmatrix}-\log_e x, 0<x<1\\ \log_e x, x\geq 1\end{Bmatrix}$

Differentiability at x =1,
LHD at x =1,
$\lim_{x \to 1^-} \frac{f(x)-f(1)}{x-1} = \lim_{h \to 0} \frac{f(1-h)-f(1)}{1-h-1}$
$= \lim_{h \to 0} \frac{\log 1-h}{-h} = -1$
RHD at x =1,
$\lim_{x \to 1^+} \frac{f(x)-f(1)}{x-1} = \lim_{h \to 0} \frac{f(1+h)-f(1)}{1+h-1}$
$= \lim_{h \to 0} \frac{\log(1+h)}{h} = 1$
So, $f'(1^+) = 1$ and $f'(1^-) = -1$

32. **(d)** 1
**Explanation:** $y = \log \sqrt{\tan x}$
$\frac{dy}{dx} = \frac{1}{\sqrt{\tan x}} \times \frac{1}{2\sqrt{\tan x}} \sec^2 x$
$\frac{dy}{dx} = \frac{\sec^2 x}{2 \tan x}$
$\left|\frac{dy}{dx}\right|_{x=\frac{\pi}{4}} = \frac{\sec^2 \frac{\pi}{4}}{\sqrt{\tan \frac{\pi}{4}}} = \frac{2}{2 \times 1} = 1$

33. **(d)** R
**Explanation:** R

34. **(c)** $-1 \leq x < \frac{1}{\sqrt{2}}$
**Explanation:** We have $\cos^{-1}x > \sin^{-1}x$
$\Rightarrow \frac{\pi}{2} - \sin^{-1} x > \sin^{-1} x$
$\Rightarrow \frac{\pi}{2} > 2 \sin^{-1} x$
$\Rightarrow \sin^{-1} x < \frac{\pi}{4}$ ... (i)
But $-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}$ .... (ii)
From (i) and (ii), $-\frac{\pi}{2} \leq \sin^{-1} x < \frac{\pi}{4}$
$\Rightarrow \sin\left(-\frac{\pi}{2}\right) \leq x < \sin \frac{\pi}{4}$
$\Rightarrow -1 \leq x < \frac{1}{\sqrt{2}}$

35. **(c)** makes an acute angle with x-axis
**Explanation:** $y = 2x^7 + 3x + 5$
$\Rightarrow \frac{dy}{dx} = 14x^6 + 3$
Even power is always positive.
Hence, $\frac{dy}{dx} > 0$
$\tan \theta > 0$
Hence, tangent makes an acute angle with x-axis to the curve.

36. **(c)** Linear constraints
**Explanation:** In a LPP, the linear inequalities or restrictions on the variables are called Linear constraints.

37. **(a)** $\frac{1}{k} . A^{-1}$
**Explanation:** by the property of inverse
$(AB)^{-1} = B^{-1}A^{-1}$
$(KA)^{-1} = A^{-1}K^{-1}$
$= \frac{1}{K} A^{-1}$

38. **(b)** null matrix
**Explanation:** Only a null matrix can be symmetric as well as skew symmetric.
In Symmetric Matrix $A^T = A$,
Skew Symmetric Matrix $A^T = -A$,

Given that the matrix is satisfying both the properties.
Therefore, Equating the RHS we get A = -A i.e, 2A = 0.
Therefore A = 0, which is a null matrix.

39. **(a)** $\frac{1}{2a}$

**Explanation:** $\sqrt{x}+\sqrt{y}=\sqrt{a}.......(1)$

$\Rightarrow \frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}\frac{dy}{dx}=0$

$\Rightarrow \frac{dy}{dx}=-\frac{\sqrt{y}}{\sqrt{x}}.......(2)$

$\Rightarrow \frac{d^2y}{dx^2}=-\frac{\sqrt{x}\frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx}-\sqrt{y}\frac{1}{2}x^{-\frac{1}{2}}}{x}$

$=\frac{-\left(\frac{\sqrt{x}}{2\sqrt{y}}\left(-\frac{\sqrt{y}}{\sqrt{x}}\right)-\frac{\sqrt{y}}{2\sqrt{x}}\right)}{x}$

$=\frac{\sqrt{x}+\sqrt{y}}{2x\sqrt{x}}=\frac{\sqrt{a}}{2x\sqrt{x}.}=\frac{\sqrt{a}}{2a\sqrt{a}}=\frac{1}{2a}$

40. **(d)** neither one-one nor onto

**Explanation:** Given that f : R $\rightarrow$ R be a function where

$f(x)=\frac{x^2-8}{x^2+2}$

Here, we can see that for negative as well as positive x we will get same value.
So, it is not one-one.

y = f(x)

$\Rightarrow \mathrm{y}=\frac{\mathrm{x}^2-8}{\mathrm{x}^2+2}$

$\Rightarrow$ y($x^2$ + 2) = ($x^2$ - 8)

$\Rightarrow$ $x^2$(y - 1) = -2y - 8

$\Rightarrow \mathrm{x}=\sqrt{\frac{2\mathrm{y}+8}{1-\mathrm{y}}}$

For y = 1 , no x is defined.
So, f is not onto.

**SECTION – C**

41. **(d)** $4a=3\beta$

**Explanation:** $\alpha=\tan^{-1}\left(\tan\frac{5\pi}{4}\right)$

$\Rightarrow \alpha=\tan^{-1}\left(\tan\left(\pi+\frac{\pi}{4}\right)\right)$

$\Rightarrow \alpha=\tan^{-1}\left(\tan\left(\frac{\pi}{4}\right)\right)$

$\Rightarrow \alpha=\frac{\pi}{4}$

and

$\beta=\tan^{-1}\left(\tan\left(\pi-\frac{2\pi}{3}\right)\right)$

$\beta=\tan^{-1}\left(\tan\left(\frac{\pi}{3}\right)\right)$

$\beta=\frac{\pi}{3}$

$4\alpha=4\times\frac{\pi}{4}=\pi$ ...(i)

$3\beta=3\times\frac{\pi}{3}=\pi$ ...(ii)

From (i) and (ii)

$4\alpha=3\beta.$

Which is the required solution.

42. **(d)** 132

**Explanation:** Here , minimize Z = 3x + 4y ,

| **Corner points** | **Z = 3x + 4y** |
|---|---|
| C( 0 ,38 ) | 132......................(Min.) |
| B ( 52 ,0) | 156 |
| D(44 , 16) | 196 |

The minimum value is 132

43. **(c)** $m^2y$

**Explanation:** $y = ae^{mx} + be^{-mx} \Rightarrow y_1 = ame^{mx} + (-m)be^{-mx} \Rightarrow y_2 = am^2e^{mx} + (m^2)be^{-mx}$
$\Rightarrow y_2 = m^2(ae^{mx} + be^{-mx}) \Rightarrow y_2 = m^2y$

44. **(b)** -1

**Explanation:** $f(x) = 2x^3 - 3x^2 - 12x + 5$

$\Rightarrow f'(x) = 6x^2 - 6x - 12$

For local maxima or minima we have

$f'(x) = 0$

$6x^2 - 6x - 12 = 0$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow x = 2$ or $x = -1$

$f''(x) = 12x - 6$

$f''(2) = 18 > 0$

function has local minima at x = 2.

$f''(-1) = -18 < 0$

function has local maxima at x = -1.

45. **(a)** an equivalence relation

**Explanation:** an equivalence relation

Reflexivity: Let $a \in R$

Then, $aa = a^2 > 0$

$\Rightarrow (a, a) \in R \forall a \in R$

So, S is reflexive on R.

Symmetry: Let $(a, b) \in S$

Then,

$(a, b) \in S$

$\Rightarrow ab \geq 0$

$\Rightarrow ba \geq 0$

$\Rightarrow (b, a) \in S \forall a, b \in R$

So, S is symmetric on R.

Transitive:

If $(a, b), (b, c) \in S$

$\Rightarrow ac \geq 0$ $[\because b2 \geq 0]$

$\Rightarrow (a, c) \in S$ for all a, b, c $\in$ set R

Hence,. S is an equivalence relation on R

46. **(d)** $\frac{1}{3}t^2 + 20t + 1$

**Explanation:** $\frac{1}{3}t^2 + 20t + 1$

47. **(c)** 376 miles/sec

**Explanation:** 376 miles/sec

48. **(d)** 27 seconds

**Explanation:** 27 seconds

49. **(a)** 21

**Explanation:** 21

50. **(c)** $\frac{4}{3}$

**Explanation:** $\frac{4}{3}$